UNIT-1

(Lecture-8)

Realization of Digital Systems: FIR Cascaded Lattice Structures

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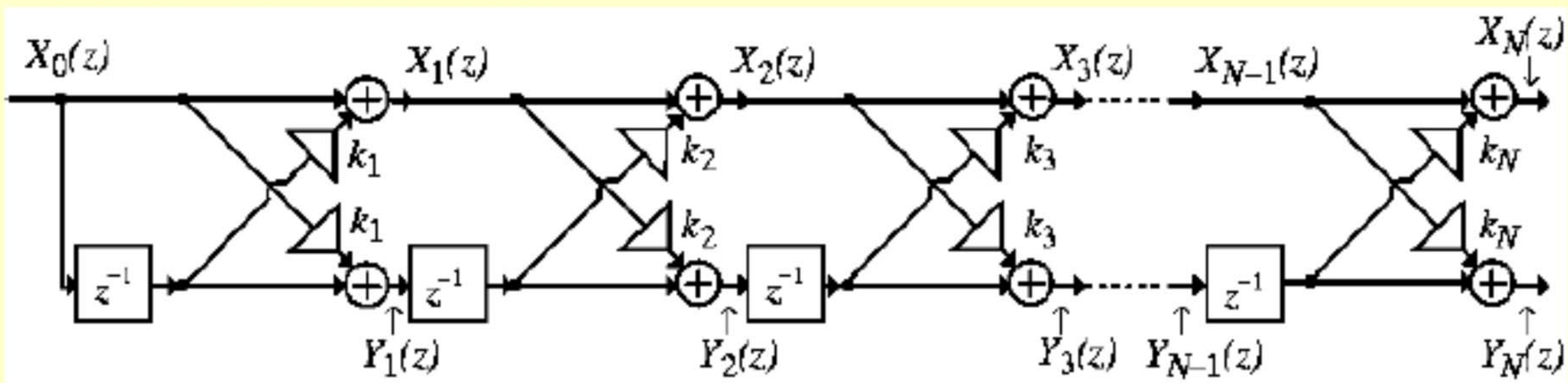
FIR Cascaded Lattice Structures

- An arbitrary Nth-order FIR transfer function of the form

$$H_N(z) = 1 + \sum_{n=1}^{N} p_n z^{-n}$$

can be realized as a cascaded lattice structure as shown below



FIR Cascaded Lattice Structures

- From figure, it follows that

$$X_m(z) = X_{m-1}(z) + k_m z^{-1} Y_{m-1}(z)$$

$$Y_m(z) = k_m X_{m-1}(z) + z^{-1} Y_{m-1}(z)$$

- In matrix form the above equations can be written as

$$\begin{bmatrix} X_m(z) \\ Y_m(z) \end{bmatrix} = \begin{bmatrix} 1 & k_m z^{-1} \\ k_m & z^{-1} \end{bmatrix} \begin{bmatrix} X_{m-1}(z) \\ Y_{m-1}(z) \end{bmatrix}$$

where $m = 1, 2, \ldots, N$

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- Denote

$$H_m(z) = \frac{X_m(z)}{X_0(z)}, \qquad G_m(z) = \frac{Y_m(z)}{X_0(z)}$$

- Then it follows from the input-output relations of the *m*-th two-pair that

$$H_m(z) = H_{m-1}(z) + k_m z^{-1} G_{m-1}(z)$$

$$G_m(z) = k_m H_{m-1}(z) + z^{-1} G_{m-1}(z)$$

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- From the previous equation we observe

$$H_1(z) = 1 + k_1 z^{-1}, \quad G_1(z) = k_1 + z^{-1}$$

where we have used the facts

$$H_0(z) = X_0(z) / X_0(z) = 1$$

$$G_0(z) = Y_0(z) / X_0(z) = X_0(z) / X_0(z) = 1$$

- It follows from the above that

$$G_1(z) = z^{-1}(z\,k_1 + 1) = z^{-1} H_1(z^{-1})$$

⟹ $G_1(z)$ is the mirror-image of $H_1(z)$

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- From the input-output relations of the m-th two-pair we obtain for $m = 2$

$$H_2(z) = H_1(z) + k_2 z^{-1} G_1(z)$$

$$G_2(z) = k_2 H_1(z) + z^{-1} G_1(z)$$

- Since $H_1(z)$ and $G_1(z)$ are 1st-order polynomials, it follows that $H_2(z)$ and $G_2(z)$ are 2nd-order polynomials

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- Substituting $G_1(z) = z^{-1}H_1(z^{-1})$ in the two previous equations we get

$$H_2(z) = H_1(z) + k_2 z^{-2} H_1(z^{-1})$$
$$G_2(z) = k_2 H_1(z) + z^{-2} H_1(z^{-1})$$

- Now we can write

$$G_2(z) = k_2 H_1(z) + z^{-2} H_1(z^{-1})$$
$$= z^{-2}[k_2 z^2 H_1(z) + H_1(z^{-1})] = z^{-2} H_2(z^{-1})$$

⇒ $G_2(z)$ is the mirror-image of $H_2(z)$

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- In the general case, from the input-output relations of the m-th two-pair we obtain

$$H_m(z) = H_{m-1}(z) + k_m z^{-1} G_{m-1}(z)$$

$$G_m(z) = k_m H_{m-1}(z) + z^{-1} G_{m-1}(z)$$

- It can be easily shown that

$$G_m(z) = z^{-m} H_m(z^{-1}), \; m = 1, 2, \ldots, N$$

➡ $G_m(z)$ is the mirror-image of $H_m(z)$

FIR Cascaded Lattice Structures

- To develop the synthesis algorithm, we express $H_{m-1}(z)$ and $G_{m-1}(z)$ in terms of $H_m(z)$ and $G_m(z)$ for $m = N, N-1, \ldots, 1$ arriving at

$$H_{N-1}(z) = \frac{1}{(1-k_N^2)}\{H_N(z) - k_N G_N(z)\}$$

$$G_{N-1}(z) = \frac{1}{(1-k_N^2)z^{-1}}\{-k_N H_N(z) + G_N(z)\}$$

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- Substituting the expressions for

$$H_N(z) = 1 + \sum_{n=1}^{N} p_n z^{-n}$$

and

$$G_N(z) = z^{-N} H_N(z^{-1}) = \sum_{n=1}^{N-1} p_n z^{-N+n} + z^{-N}$$

in the first equation we get

$$H_{N-1}(z) = \frac{1}{1-k_N^2}\{(1-k_N p_N) + \sum_{n=1}^{N-1}(p_n - k_N p_{N-n}) z^{-n} + (p_N - k_N) z^{-N}\}$$

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- If we choose $k_N = p_N$, then $H_{N-1}(z)$ reduces to an FIR transfer function of order $N-1$ and can be written in the form

$$H_{N-1}(z) = 1 + \sum_{n=1}^{N-1} p'_n z^{-n}$$

where

$$p'_n = \frac{p_n - k_N p_{N-n}}{1 - k_N^2}, 1 \le n \le N-1$$

- Continuing the above recursion algorithm, all multiplier coefficients of the cascaded lattice structure can be computed

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- Example: Realize the FIR transfer function

$$H_4(z) = 1 + 1.2z^{-1} + 1.12z^{-2} + 0.12z^{-3} - 0.08z^{-4}$$

From the above, we observe $k_4 = p_4 = -0.08$

and using $p'_n = \frac{p_n - k_4 p_{4-n}}{1-k_4^2},\ 1 \le n \le 3$

we determine the coefficients of $H_3(z)$ as

$$p'_3 = 0.2173913,\ p'_2 = 1.2173913$$

$$p'_1 = 1.2173913$$

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- As a result,

$$H_3(z) = 1 + 1.2173913\, z^{-1} + 1.2173913\, z^{-2} + 0.2173913\, z^{-3}$$

- Thus, $k_3 = p'_3 = 0.2173913$
- Using

$$p''_n = \frac{p'_n - k_3 p'_{2-n}}{1 - k_3^2},\ 1 \le n \le 2$$

we determine the coefficients of $H_2(z)$ as

$$p''_2 = 1.0, \qquad p''_1 = 1.0$$

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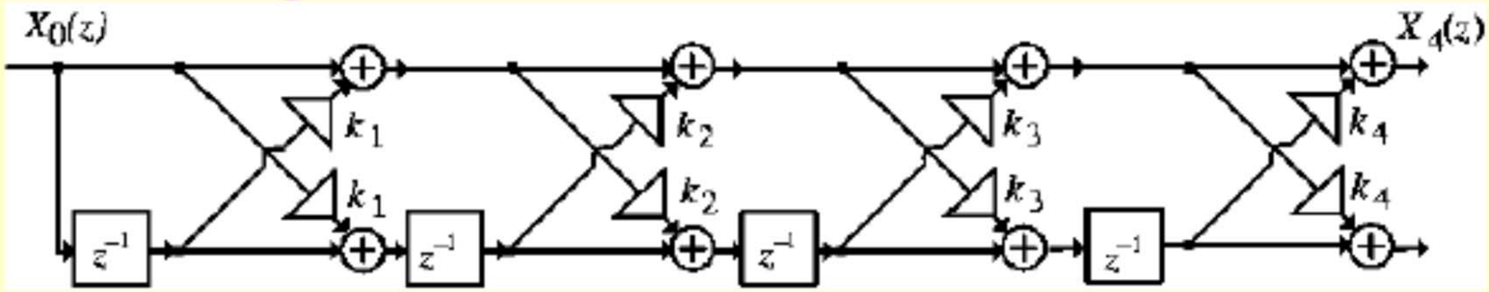
- As a result, $H_2(z) = 1 + z^{-1} + z^{-2}$
- From the above, we get $k_2 = p_2'' = 1$
- The last recursion yields the last multiplier coefficient $k_1 = p_1''/(1 + k_2) = 0.5$
- The complete realization is shown below



$$k_1 = 0.5,\ k_2 = 1,\ k_3 = 0.2173913,\ k_4 = -0.08$$